# A partial version of Dynamic Epistemic Logic

**Jens Ulrik Hansen**
Department of Philosophy,
Lund University, Sweden

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# Motivation

**Epistemic modeling and Dynamic Epistemic Logic**

- (Dynamic) Epistemic Logic as a formal mathematical framework for modeling of epistemic scenarios
- "Possible worlds" models, key idea: *Knowledge = true in all conceivable alternative situations*
- Dynamic Epistemic Logic (DEL) provides a nice and general theory about how possible worlds models can evolve.
- However, not all forms of dynamics can be captured by the DEL framework
- Especially, new agents entering and new (atomic) fact becoming relevant cannot be captured by the DEL framework

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# Outline

- A brief introduction to Dynamic epistemic logic (DEL)
- The shortcomings of DEL
- Partial possible world models
- A partial version of DEL
- Reduction axioms for Partial DEL
- Concluding remarks and future research

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**Definition (Possible worlds models)**

Given a set of agents $\mathbb{A}$ and a set of propositional variables PROP, a possible worlds model is a tuple $\mathcal{M} = \langle W, (R_a)_{a\in\mathbb{A}}, V\rangle$, where $W$ is a non-empty set of states/worlds, $R_a$ is a binary (equivalence) relation on $W$ (for each $a \in \mathbb{A}$), and $V : W \times \text{PROP} \to \{0, 1\}$ is a valuation.

**A (typical) epistemic scenario**

- *Three friends, Arnold (a), Bruce (b), and Chuck (c) are having a friendly game of Texas hold 'em poker at Bruce's place. Arnold shuffles the cards and deals them...*

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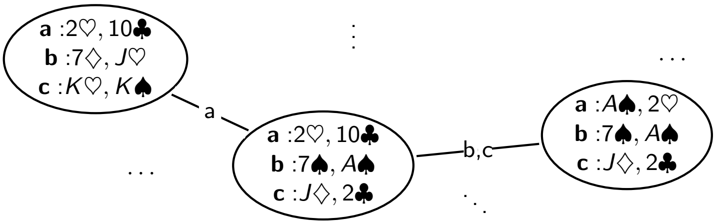

# A brief introduction to Dynamic epistemic logic

**Definition (Syntax of epistemic logic)**

Given a set of agents $\mathbb{A}$ and a set of propositional variables PROP, the language of epistemic logic is given by

$$\varphi \quad ::= \quad p \mid \top \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \quad ,$$

where $p \in \text{PROP}$ and $a \in \mathbb{A}$.

**Definition (Semantics of epistemic logic)**

Given a model $\mathcal{M} = \langle W, (R_a)_{a\in\mathbb{A}}, V\rangle$, a $w \in W$, and a formula $\varphi$, the semantics is given by

$$\begin{array}{lll}
\mathcal{M}, w \models p & \text{iff} & V(w,p) = 1 \\
\mathcal{M}, w \models \top & & \\
\mathcal{M}, w \models \neg\varphi & \text{iff} & \mathcal{M}, w \not\models \varphi \\
\mathcal{M}, w \models \varphi \wedge \psi & \text{iff} & \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi \\
\mathcal{M}, w \models K_a\varphi & \text{iff} & \text{for all } v \in V : R_a(w,v) \text{ implies that } \mathcal{M}, v \models \varphi
\end{array}$$

**...the example continued...**

- *The betting begins with no one folding, after which Chuck, being the dealer, deals the flop (puts three cards face-up on the table). Assume J♡ is one of them.*

  *– Any world where a player holds J♡ is no longer possible.*

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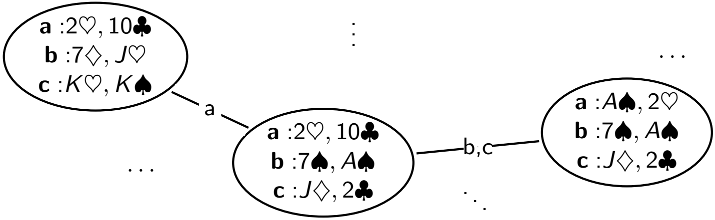

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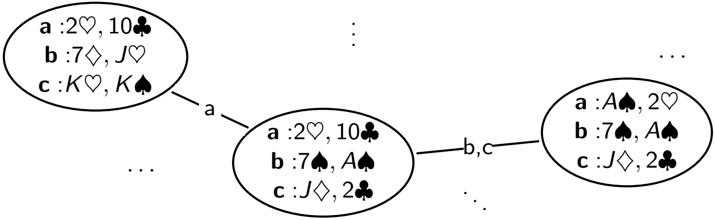

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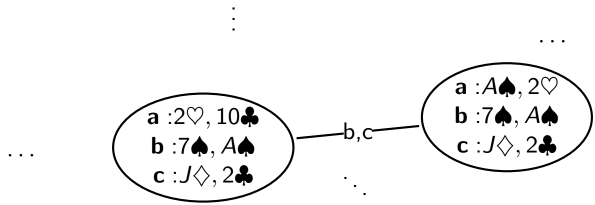

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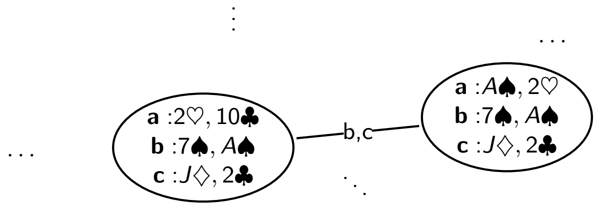


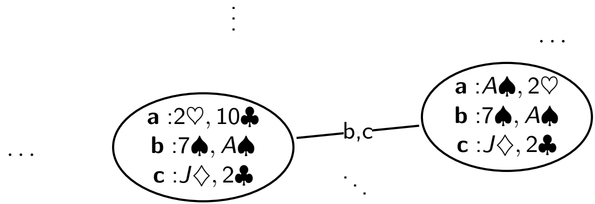

**Public Announcement Logic**

- The simplest form of dynamic epistemic logic
- A public announcement of $\varphi$ deletes all worlds where $\varphi$ was true
- $[\varphi]\psi$ reads "after the public announcement of $\varphi$, $\psi$ is the case

**...the example continued...**

- *Now, Arnold shows one of his cards (10♣) to Bruce.*

  *– All links from the actual world to any world where Arnold does not hold 10♣ should be removed.*



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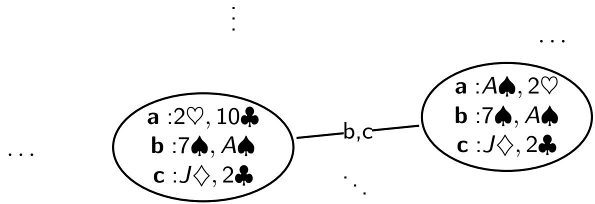

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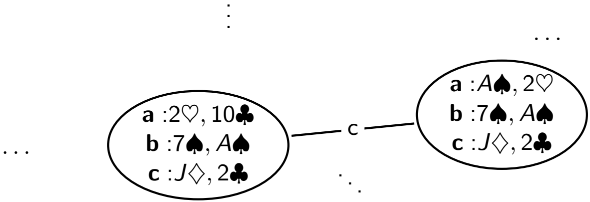

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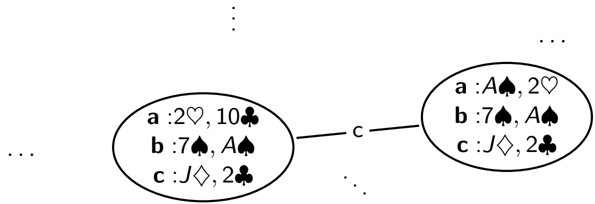


- To model this kind of dynamics, Dynamic Epistemic Logic introduces action (or event) models.
- With event models we can model more complex dynamics such as removing relational links or even expanding the model...

# A brief introduction to Dynamic epistemic logic

## Definition (Event model)

An event model is atuple $\mathcal{E} = \langle E, (S_a)_{a \in \mathbb{A}}, \text{pre} \rangle$, where $E$ is a set of possible events, $S_a$ is an equivalence relation for each $a$, and $\text{pre} : E \to \mathcal{L}$ is function assigning a precondition formula $\text{pre}(e)$ to each event $e$.

## Definition (Product update)

For an epistemic model $\mathcal{M} = \langle W, R_a, V \rangle$ and an event model $\mathcal{E} = \langle E, S_a, \text{pre} \rangle$, the product $\mathcal{M} \otimes \mathcal{E} = \langle W', R'_a, V' \rangle$ is defined by

- $W' = \{(w, e) \in W \times E \mid \mathcal{M}, w \models \text{pre}(e)\}$
- $(w, e) R'_a (v, f)$ iff $w R_a v$ and $e S_a f$
- $V((w, e), p) = V(w, p)$.

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**Definition (Syntax)**

For each event model $\mathcal{E}$, event $e$ of $\mathcal{E}$, and formula $\varphi$, $[\mathcal{E}, e]\varphi$ is also a formula of the language.

**Definition (Semantics)**

The semantics of the formula $[\mathcal{E}, e]\varphi$ is defined by:

$$\mathcal{M}, w \models [\mathcal{E}, e]\varphi \quad \text{iff} \quad \mathcal{M}, w \models \text{pre}(e) \Rightarrow \mathcal{M} \otimes \mathcal{E}, (w, e) \models \varphi$$

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**Reduction axioms for DEL:**

The modality $[\mathcal{E}, e]$ can be reduced away using the following validities:

$$
\begin{aligned}
[\mathcal{E}, e]p &\leftrightarrow \mathsf{pre}(e) \to p \\
[\mathcal{E}, e](\varphi \wedge \psi) &\leftrightarrow [\mathcal{E}, e]\varphi \wedge [\mathcal{E}, e]\psi \\
[\mathcal{E}, e]\neg\varphi &\leftrightarrow \mathsf{pre}(e) \to \neg[\mathcal{E}, e]\varphi \\
[\mathcal{E}, e]K_a\varphi &\leftrightarrow \mathsf{pre}(e) \to \bigwedge_{f \in E, eS_a f} K_a[\mathcal{E}, f]\varphi,
\end{aligned}
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- *Now, assume that Bruce's wife walks in. One could easily imagine that Bruce and his wife have a secret way of communicating and that Bruce's wife is able to see chuck's cards.*
- How to change the model will depend on what we assume about what Bruce's wife knows and what Arnold, Bruce, and Chuck knows about her knowledge, whether she communicated Chuck's cards to Bruce...and so on

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- How to change the model will depend on what we assume about what Bruce's wife knows and what Arnold, Bruce, and Chuck knows about her knowledge, whether she communicated Chuck's cards to Bruce...and so on
- *Event models only let us deal with agents already mentioned in both the epistemic model and the event model.*

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**...the example continued...**

- *Now assume that Arnold starts to tell a story about how he used to cheat as a dealer in Las Vegas. This of course, raises the question of whether Arnold cheated when he dealt the cards.*
- Thus, there is a new atomic proposition "Arnold cheated when dealing the cards" that Bruce and Chuck are uncertain about.

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- Thus, there is a new atomic proposition "Arnold cheated when dealing the cards" that Bruce and Chuck are uncertain about.
- *In standard DEL we needed to have included this propositional variable from the beginning to capture the proper modeling.*

**Why we need a partial version of DEL**

- Standard Dynamic Epistemic Logic is not quite powerful enough to capture all kinds of natural dynamics!
- Enhanced modeling of epistemic scenarios
- Get rid of the "closed-world" assumption in epistemic modeling
- A more "economical" representation of ignorance/absence of knowledge.
- More intuitive: Does it follows from that an agent does not consider a world possible where $P$ is true that he knows $\neg P$?!
- Logical Omniscience is "less" of a problem
- Another approach to awareness
- Partial models seem natural from a modeling perspective, so why not make them full-blown citizen and develop a language to talk about them?

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**Syntax:**

We will use standard epistemic language to talk about these models:

$$\varphi ::= p \mid \top \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \,,$$

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**The partial semantics of the language:**

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\begin{array}{lll}
\mathcal{M}, w \models p & \text{iff} & V(w,p) = 1 \\
\mathcal{M}, w \mathrel{=\!\!|} p & \text{iff} & V(w,p) = 0 \\
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\mathcal{M}, w \models \top & & \\
\mathcal{M}, w \not\mathrel{=\!\!|} \top & & \\
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\mathcal{M}, w \models \varphi \wedge \psi & \text{iff} & \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi \\
\mathcal{M}, w \mathrel{=\!\!|} \varphi \wedge \psi & \text{iff} & \mathcal{M}, w \mathrel{=\!\!|} \varphi \text{ or } \mathcal{M}, w \mathrel{=\!\!|} \psi \\
\\
\mathcal{M}, w \models K_a\varphi & \text{iff} & \forall v \in W; wR_a v \Rightarrow \mathcal{M}, v \models \varphi \\
\mathcal{M}, w \mathrel{=\!\!|} K_a\varphi & \text{iff} & \exists v \in W; wR_a v \text{ and } \mathcal{M}, v \mathrel{=\!\!|} \varphi
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This logic is essentially just stand partial modal logic (Jaspars 1994, Jaspars and Thijsse 1996)

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| $\mathcal{M}, w \models \neg\varphi$ | iff | $\mathcal{M}, w \dashv \varphi$ |
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| $\mathcal{M}, w \models \varphi \wedge \psi$ | iff | $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$ |
| $\mathcal{M}, w \dashv \varphi \wedge \psi$ | iff | $\mathcal{M}, w \dashv \varphi$ or $\mathcal{M}, w \dashv \psi$ |
| $\mathcal{M}, w \models K_a\varphi$ | iff | $\forall v \in W; wR_av \Rightarrow \mathcal{M}, v \models \varphi$ |
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## A minor issue

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- Given a partial model $\mathcal{M} = \langle W, A, P, (R_a)_{a \in A}, V \rangle$, what happens to clauses for $K_a$ when $a \notin A$?
- There are several options one could take...
- We take the route of assuming that agents not in the model will know nothing more than what is common knowledge to all the agents in the scenario.
- Given a partial model $\mathcal{M} = \langle W, A, P, (R_a)_{a \in A}, V \rangle$ and an $a \notin A$, we will let $R_a$ denote $W \times W$, when evaluating formulas of the form $K_a\varphi$ on $\mathcal{M}$ and keep the semantic clauses for $K_a$.
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- A similar issue for $p \notin P$ is taken care of by partiality

## A minor issue

$$\begin{array}{lll} \mathcal{M}, w \models K_a\varphi & \text{iff} & \forall v \in W; wR_a v \Rightarrow \mathcal{M}, v \models \varphi \\ \mathcal{M}, w \mathrel{=\!\!|} K_a\varphi & \text{iff} & \exists v \in W; wR_a v \text{ and } \mathcal{M}, v \mathrel{=\!\!|} \varphi \end{array}$$

- Given a partial model $\mathcal{M} = \langle W, A, P, (R_a)_{a \in A}, V \rangle$, what happens to clauses for $K_a$ when $a \notin A$?
- There are several options one could take...
- We take the route of assuming that agents not in the model will know nothing more than what is common knowledge to all the agents in the scenario.
- Given a partial model $\mathcal{M} = \langle W, A, P, (R_a)_{a \in A}, V \rangle$ and an $a \notin A$, we will let $R_a$ denote $W \times W$, when evaluating formulas of the form $K_a\varphi$ on $\mathcal{M}$ and keep the semantic clauses for $K_a$.
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# Outline

- A brief introduction to Dynamic epistemic logic (DEL)
- The shortcomings of DEL
- Partial possible world models
- A partial version of DEL
- Reduction axioms for Partial DEL
- Concluding remarks and future research

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# A Partial version of DEL

**Definition (Substitution)**

A substitution is a function $\sigma : X \to \mathcal{L}$, where $X$ is a finite set of propositional variables and $\mathcal{L}$ is language. $X$ is the domain of $\sigma$ (denoted $dom(\sigma)$). For $\mathcal{L}$ and $P$, the set of all substitutions $\sigma : X \to \mathcal{L}$, where $X \subseteq P$ will be denoted $sub(P, \mathcal{L})$.

**Definition (Event model)**

An (partial) event model is a tuple $\mathcal{E} = \langle E, B, Q, (S_a)_{a \in B}, \mathrm{pre}, \mathrm{post} \rangle$, s.t.

- $E$ is non-empty set of events
- $B$ is a set of agents
- $Q$ is a set of propositional variables
- $S_a$ is an equivalence relation for each $a \in B$
- $\mathrm{pre} : E \to \mathcal{L}(B, Q)$ is a precondition function
- $\mathrm{post} : E \to sub(Q, \mathcal{L}(B, Q))$ is a postcondition function specifying what propositional variables will change if an event happens.

# A Partial version of DEL

## Definition (Product update)

Given $\mathcal{M} = \langle W, A, P, (R_a)_{a \in A}, V \rangle$ and $\mathcal{E} = \langle E, B, Q, (S_a)_{a \in B}, \text{pre}, \text{post} \rangle$, the product update $\mathcal{M} \otimes \mathcal{E} = \langle W', A', P', (R'_a)_{a \in A'}, V' \rangle$ is:

- $W' = \{(w, e) \in W \times E \mid \mathcal{M}, w \models \text{pre}(e)\}$
- $A' = B$
- $P' = Q$
- $(w, e) R'_a (v, f)$ iff
  - $wR_a v$ and $eS_a f$ , if $a \in B \cap A$, and
  - $eR_a f$ , if $a \in B \setminus A$
- $V'((w, e), p) =$
  - 1, if $p \in Q$ and $\mathcal{M}, w \models \text{post}(e)(p)$, and
  - 0, if $p \in Q$ and $\mathcal{M}, w \mathrel{=\!\!|} \text{post}(e)(p)$

- Convention: $\text{post}(e)(p) = p$ if $p \in Q \setminus dom(\text{post}(e))$ (and $\text{post}(e)(p) = \bot$ if $p \notin Q$).
- Note: if $p \in Q \setminus dom(\text{post}(e))$ then the value of $p$ at $(w, e)$, $V'((w, e), p)$, becomes $V(w, p)$ (as in standard DEL).

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**Semantics:**

$$\mathcal{M}, w \models [\mathcal{E}, e]\varphi \quad \text{iff} \quad \mathcal{M}, w \models \text{pre}(e) \text{ implies } \mathcal{M} \otimes \mathcal{E}, (w, e) \models \varphi$$

$$\mathcal{M}, w \mathrel{=\!\!|} [\mathcal{E}, e]\varphi \quad \text{iff} \quad \mathcal{M}, w \models \text{pre}(e) \text{ and } \mathcal{M} \otimes \mathcal{E}, (w, e) \mathrel{=\!\!|} \varphi$$

The resulting logic will be referred to as *Partial Dynamic Epistemic Logic (ParDEL)*.

**Alternative choices for the semantics:**

- A possible alternative:

$$\mathcal{M}, w \models [\mathcal{E}, e]\varphi \quad \text{iff} \quad \mathcal{M}, w \mathrel{=\!\!|} \text{pre}(e) \text{ or } \mathcal{M} \otimes \mathcal{E}, (w, e) \models \varphi$$

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**The example revisited**

- Adding the extra propositional variable "Arnold cheated when dealing the cards" ($ac$): $\mathcal{E} = \langle E, B, Q, (S_a)_{a \in B}, \mathrm{pre}, \mathrm{post} \rangle$, s.t.
  - $E = \{e_1, e_2\}$, $B = A$, $Q = P \cup \{ac\}$
  - $S_a = \{(e_1, e_1), (e_2, e_2)\}$, $S_b = S_c = E \times E$
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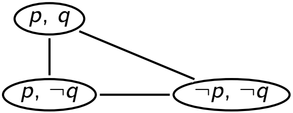
Assume $q \rightarrow p$ is true (if I get a cup of coffee, I'll get my morning shot of caffeine.)



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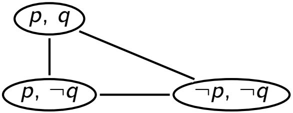


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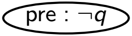


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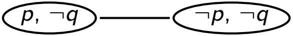

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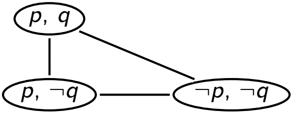


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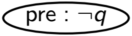


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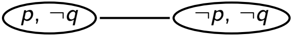
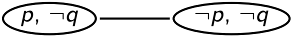



What now?!

# A Partial version of DEL



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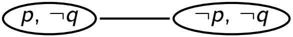


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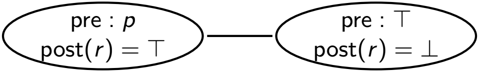

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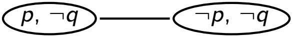


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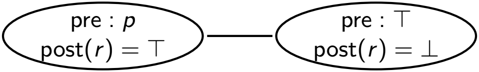


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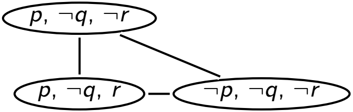

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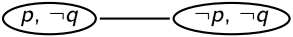


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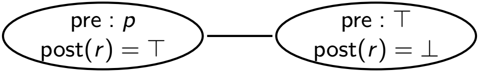


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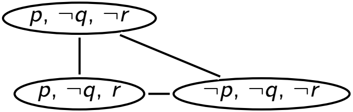


Now learning $r$ will satisfy my caffeine need!

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# Reduction axioms for Partial DEL

**Problems:**

- The metalanguage implication cannot be expressed in the logic (it's not equivalent to $\neg\varphi \vee \psi$ as $\vee, \neg$ is not functional complete in partial logic)

$$\mathcal{M}, w \models [\mathcal{E}, e]\varphi \quad \text{iff} \quad \mathcal{M}, w \models \text{pre}(e) \text{ implies } \mathcal{M} \otimes \mathcal{E}, (w, e) \models \varphi$$

- The are problems with the formula $[\mathcal{E}, e]K_a\varphi$ when $a$ is not present in the event model $\mathcal{E}$

**A solution:**

- Extend the language:
  - Add a classical negation, $\sim$, with semantics:

$$\begin{array}{lll} \mathcal{M}, w \models \sim\varphi & \text{iff} & \mathcal{M}, w \not\models \varphi \\ \mathcal{M}, w \dashv \sim\varphi & \text{iff} & \mathcal{M}, w \models \varphi \end{array}$$

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| $\mathcal{M}, w \models [\mathcal{E}, e]\top$ | | |
| $\mathcal{M}, w \models [\mathcal{E}, e]\neg\varphi$ | iff | $\mathcal{M}, w \models \text{pre}(e) \to \neg[\mathcal{E}, e]\varphi$ |
| $\mathcal{M}, w \models [\mathcal{E}, e]{\sim}\varphi$ | iff | $\mathcal{M}, w \models \text{pre}(e) \to {\sim}[\mathcal{E}, e]\varphi$ |
| $\mathcal{M}, w \models [\mathcal{E}, e](\varphi \wedge \psi)$ | iff | $\mathcal{M}, w \models [\mathcal{E}, e]\varphi \wedge [\mathcal{E}, e]\psi$ |
| $\mathcal{M}, w \models [\mathcal{E}, e]U\varphi$ | iff | $\mathcal{M}, w \models \text{pre}(e) \to \bigwedge_{f \in E} U[\mathcal{E}, f]\varphi$ |
| $\mathcal{M}, w \models [\mathcal{E}, e]K_a\varphi$ | iff | $\mathcal{M}, w \models \text{pre}(e) \to \bigwedge_{f \in X} \Box[\mathcal{E}, f]\varphi$ [1] |
| $\mathcal{M}, w \dashv [\mathcal{E}, e]p$ | iff | $\mathcal{M}, w \dashv \neg\text{pre}(e) \vee \text{post}(e)(p)$ |
| $\mathcal{M}, w \not\dashv [\mathcal{E}, e]\top$ | | |
| $\mathcal{M}, w \dashv [\mathcal{E}, e]\neg\varphi$ | iff | $\mathcal{M}, w \dashv \neg\text{pre}(e) \vee \neg[\mathcal{E}, e]\varphi$ |
| $\mathcal{M}, w \dashv [\mathcal{E}, e]{\sim}\varphi$ | iff | $\mathcal{M}, w \dashv \neg\text{pre}(e) \vee {\sim}[\mathcal{E}, e]\varphi$ |
| $\mathcal{M}, w \dashv [\mathcal{E}, e](\varphi \wedge \psi)$ | iff | $\mathcal{M}, w \dashv [\mathcal{E}, e]\varphi \wedge [\mathcal{E}, e]\psi$ |
| $\mathcal{M}, w \dashv [\mathcal{E}, e]U\varphi$ | iff | $\mathcal{M}, w \dashv \neg\text{pre}(e) \vee \bigwedge_{f \in E} U[\mathcal{E}, f]\varphi$ |
| $\mathcal{M}, w \dashv [\mathcal{E}, e]K_a\varphi$ | iff | $\mathcal{M}, w \dashv \neg\text{pre}(e) \vee \bigwedge_{f \in X} \Box[\mathcal{E}, f]\varphi$ [1] |

[1] If $a \in B \cap A$, then $X = \{f \in E \mid eS_af\}$ and $\Box = K_a$. If $a \in B \setminus A$, then $X = \{f \in E \mid eS_af\}$ and $\Box = U$. If $a \notin B$, then $X = E$ and $\Box = U$.

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- The shortcomings of DEL
- Partial possible world models
- A partial version of DEL
- Reduction axioms for Partial DEL
- Concluding remarks and future research

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# Concluding remarks and future research

**Summary:**

- A partial versions of DEL is quite natural from a (semantic) modeling perspective
- Action models are a natural framework to deal with extensions of partial models
- Partial semantics for modal logic extends
- Simple reduction axioms can be found for ParDEL as well

# Concluding remarks and future research

**Future research:**

- Partial modal logic can be translated into classical modal logic (using two translations) – can ParDEL be translated into DEL in similar manners?
- ParDEL seems like a plausible alternative to Awareness logic, what are the exact relations?
- Can ParDEL provide a new perspective on Logical Omniscience and ignorance?
- What does the proof theory of ParDEL looks like?
- What is the relationship to epistemic planning?

Thank you!